Consumer Theory Concepts

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ECON 306

Consumer's Constrained Optimization

- **Constrained optimization** (in general) always involves the following three elements:
  1. **Choose:** <some alternative>
  2. **In order to maximize:** <some objective>
  3. **Subject to**: <some constraints>
- The **Consumer's (constrained optimization) problem** is:
  1. **Choose:** <bundle of goods>
  2. **In order to maximize:** <utility>
  3. **Subject to**: <income and market prices>

Choices

- Consumers choose bundles of goods:
$$(x, y)$$
where x = amount of good x, and y = amount of good y

Constraints: The Budget Constraint

- **Budget set**: the set of all bundles of goods that are *affordable*:
$$p_x x + p_y y \leq m$$
  - Consumers can buy bundles that do not spend all income (income leftover)
- **Budget constraint**: the set of all bundles of goods that *spend all income*
$$p_x x + p_y y = m$$
  - To graph, solve for y:
$$y = \frac{m}{p_y} - \frac{p_x}{p_y}x$$

    * Vertical intercept: $\dfrac{m}{p_y}$
    * Horizontal intercept: $\dfrac{m}{p_x}$
    * Slope: $-\dfrac{p_x}{p_y}$

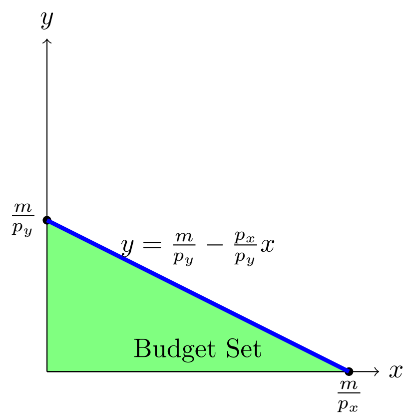


Figure 1: The Budget Constraint (blue) and Budget Set (green)

- All points on the line spend all income
  - All points beneath line are *affordable* (in budget set) but do not spend all income
  - All points above the line are *not* affordable at current income and prices
- Budget constraint determined by three parameters: p_x, p_y, m
  - Change in income: shifts budget constraint in parallel
    * New m' in intercepts
    * No change in slope
  - Change in a market price: rotates budget constraint
    * New intercept for good that changed in price
    * New slope
- Slope of budget constraint measures the *market* exchange rate between x and y (their relative prices)

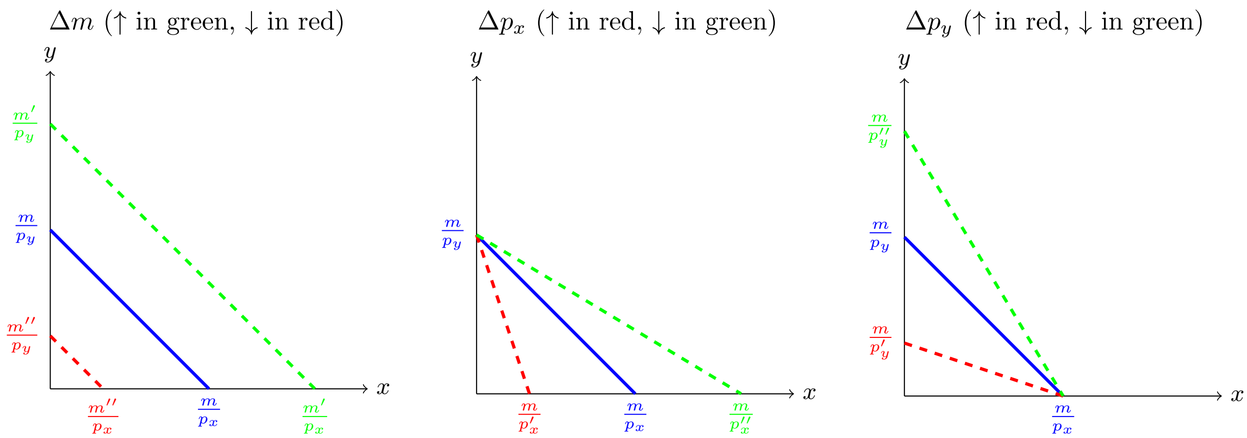


Table 1: How the budget constraint changes with income and market prices

Objective: Utility and Preferences

- **Preferences** express rankings between bundles of goods
  - For any two bundles of goods a and b:
    - $a \succ b$: a is preferred to b
    - $a \prec b$: b is preferred to a
    - $a \sim b$: indifferent between a and b
  - Assumptions about "well-behaved" preferences:
    1. Reflexivity: $a \succeq a$
    2. Completeness: for all a and b: $a \succ b$, $a \prec b$, or $a \sim b$
    3. Transitivity: if $a \succ b$ and $b \succ c \implies a \succ c$
- **Indifference curves** link all bundles which the consumer is indifferent between

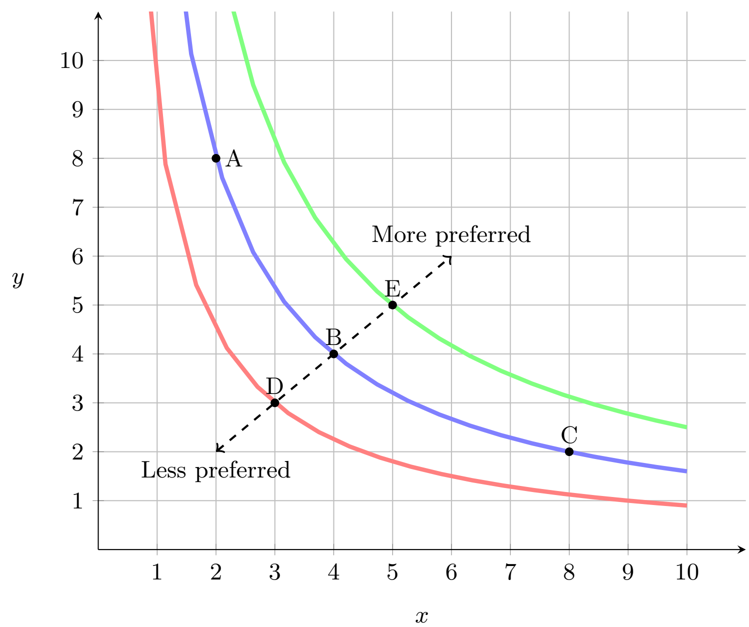


Figure 2: Indifference curves: $E \succ A \sim B \sim C \succ D$

  - Assumptions of "well-behaved" indifference curves:
    1. We can always draw indifference curves
    2. Monotonicity: "more is preferred to less"
    3. Convexity: "averages are preferred to extremes"
    4. Transitivity: indifference curves can never cross

– In general, even non-monotonic indifference curves (i.e. when there is 1 or more **bads**) follow a pattern. Figure 3 shows four types of indifference curves, broken down into four quadrants. Black arrows show the direction of *better* bundles in each of the four cases:

  I. x is a good, y is a bad
  II. x and y are both bads
  III. x and y are both goods
  IV. x is a bad, y is a good

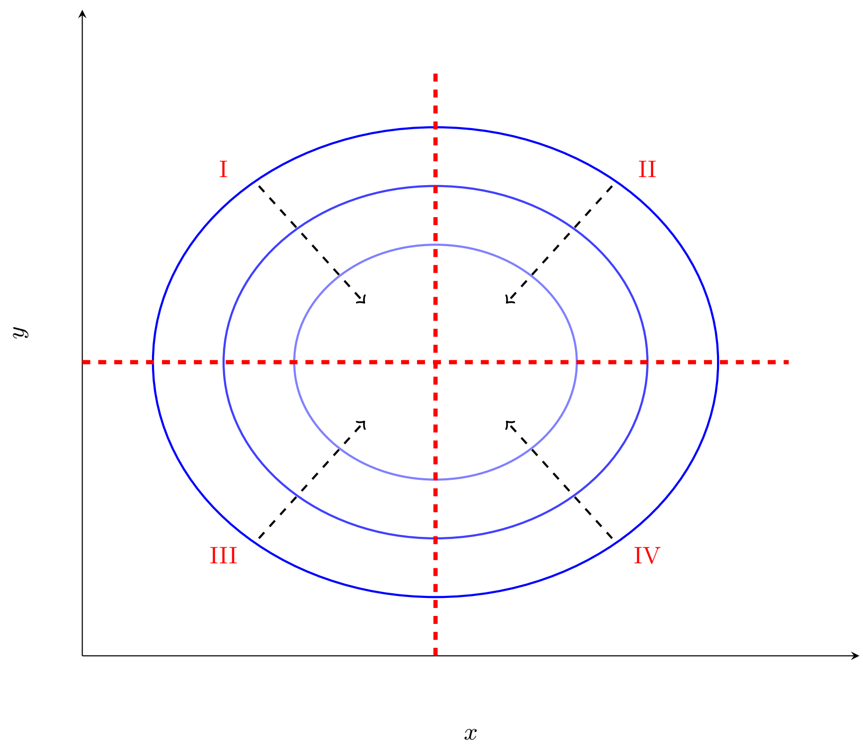


Figure 3: Possible indifference curves with goods and bads. Arrows show direction of higher utility for each quadrant.

– **Marginal rate of substitution (MRS)**: an individual's exchange rate between good x and y
  * $MRS =$ the slope of the indifference curve
  * Literally: the amount of y given up to obtain 1 more x and remain indifferent

- **Utility function**: represents preferences in functional form

$$u(x, y)$$

– We can assign utility levels to any bundles such that for any bundles a and b:

$$a \succ b \iff u(a) > u(b)$$

– Utility is **ordinal** not **cardinal**!
  * The actual utility numbers for bundle a and b mean nothing literally!
  * All that matters is if $u(a) > u(b)$, the consumer prefers a over b (we can't say *how much*)
  * Implies that multiple utility functions can represent the same preferences

– All points on the same indifference curve yield the same utility

– **Marginal utility**: the change in utility from a 1-unit increase in consumption of a good

$$MU_x = \frac{\Delta u(x,y)}{\Delta x}$$
$$MU_y = \frac{\Delta u(x,y)}{\Delta y}$$

  ∗ Marginal utilities are related to the MRS:

$$MRS = \frac{MU_x}{MU_y}$$

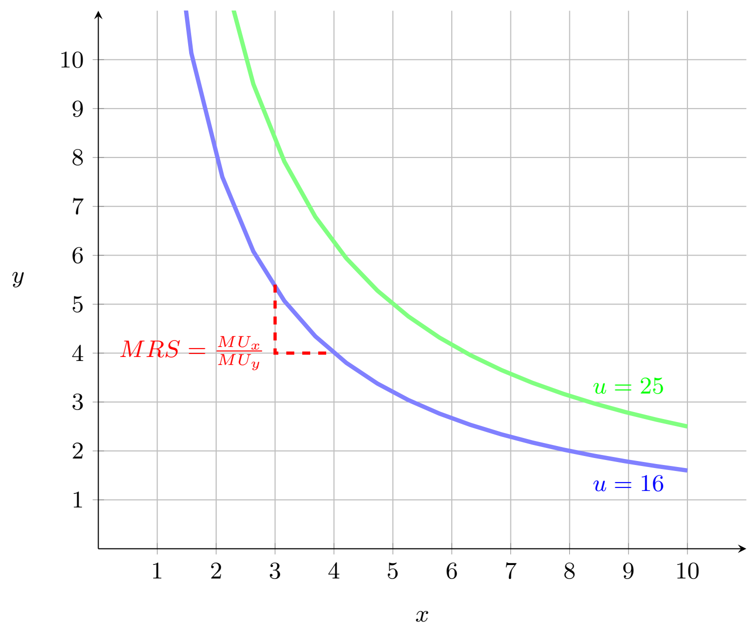
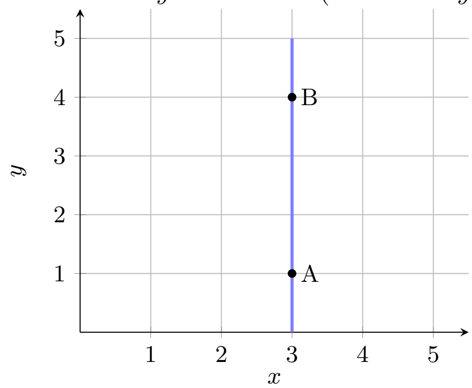


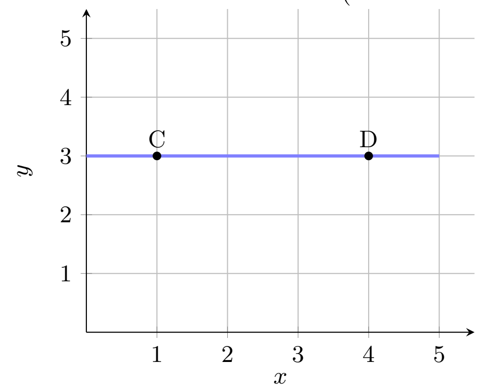
Figure 4: Indifference curves for $u(x,y) = xy$

- Shape & slopes (MRS) of indifference curves:
  * Steep vs. flat $\implies$ relative intensity of preference for x vs. y

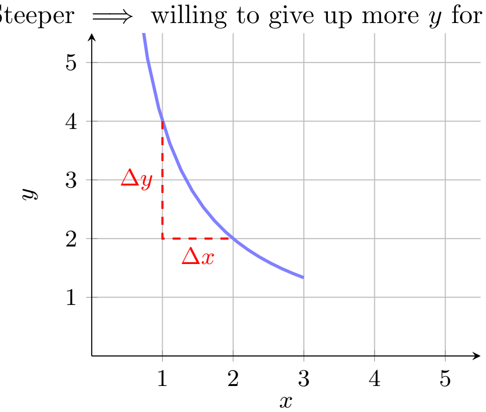
Vertical $\implies$ y is a neutral (more$\sim$less y)



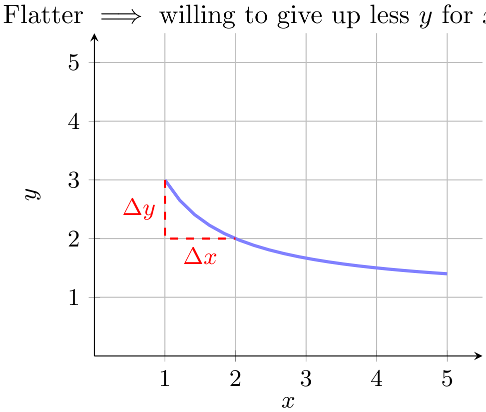
Horizontal $\implies$ x is a neutral (more$\sim$less x)



Steeper $\implies$ willing to give up more y for x

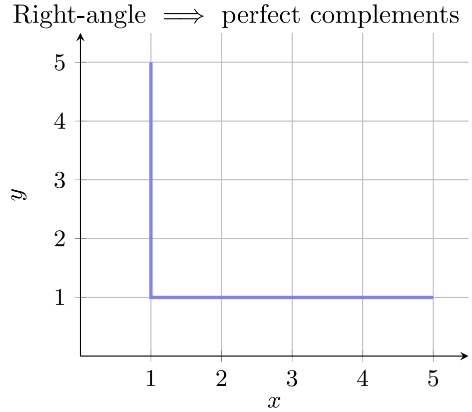


Flatter $\implies$ willing to give up less y for x



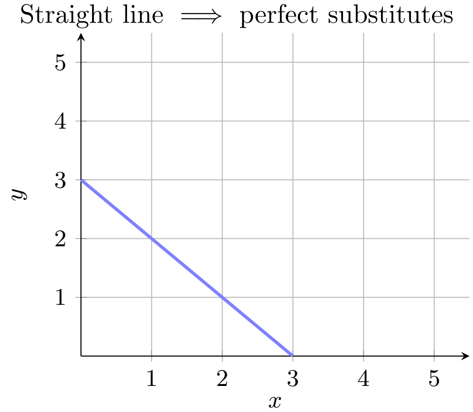
  * Bent vs. straight $\implies$ complementarity vs. substitutability between x and y

Right-angle $\implies$ perfect complements



Always consume at same rate of combination

Straight line $\implies$ perfect substitutes



Always substitute at same rate

Solving the Consumer's Problem

– Consumer chooses bundle of x and y to maximize utility subject to their income and market prices

* Expressed mathematically:

$$\max_{x,y} u(x,y)$$
$$\text{s. t. } p_x x + p_y y = m$$

* Graphically: optimum is the point of tangency between the highest indifference curve and the budget constraint

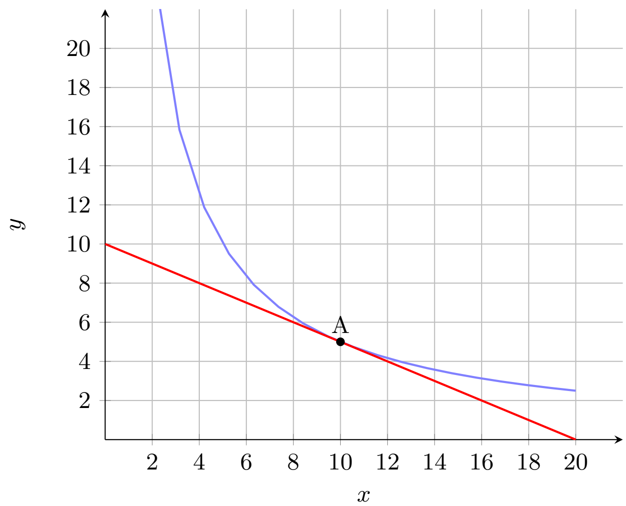


Figure 5: The consumer's optimum at point A: indifference curve is tangent to budget constraint

* At the tangency point (A), all of the following are true:

$\lvert\text{Slope of I.C.}\rvert = \lvert\text{Slope of B.C.}\rvert$	Slopes are equal
$MRS = \dfrac{p_x}{p_y}$	Definition of each slope
$\dfrac{MU_x}{MU_y} = \dfrac{p_x}{p_y}$	Individual exchange rate same as market exchange rate
$\dfrac{MU_x}{p_x} = \dfrac{MU_y}{p_y}$	Marginal utility per \$1 is the same between x and y

– **Equimarginal principle**: utility is optimized when individual can get no more utility by spending \$1 more on either x or y

* Consumer is indifferent between buying more x or buying more y: has no reason to change consumption decisions!
* If marginal utility per dollar were greater for (e.g.) x than for y, could buy more x and get more utility!

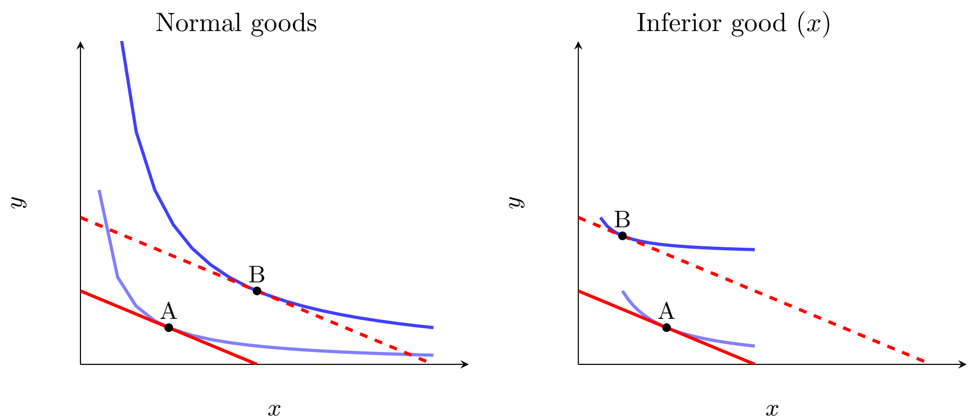
Deriving Demand

- An individual's **Demand (for good x)** is the optimal quantity that the individual would consume given current market prices and income:

$$q = D(p_x, p_y, m)$$

  We explore how a person's demand changes as one of the parameters to the demand function changes:

- Income effects $\left(\dfrac{\Delta q}{\Delta m}\right)$: how demand changes with income



Consume more x and y with $\uparrow$ m — Consume less x, more y with $\uparrow$ m

  - **Income Elasticity of Demand**: how responsive consumption is to changes in income

$$\epsilon_{q,m} = \frac{\%\Delta q}{\%\Delta m} = \frac{\left(\frac{(q_2 - q_1)}{q_1}\right)}{\left(\frac{(m_2 - m_1)}{m_1}\right)}$$

    * Measures the % change in quantity consumed for a 1% change in income
      · i.e. "if income changes by 1%, quantity consumed changes by $\epsilon_{q,m}$%"
    * If $\epsilon > 0$: **normal good**: consume more with higher income (and vice versa)
      · If $0 < \epsilon < 1$: **necessity**: increase consumption by proportionately less than income increase
      · If $\epsilon > 1$: **luxury**: increase consumption by proportionately more than income increase
    * If $\epsilon < 0$: **inferior good**: consume less with higher income (and vice versa)

- Price effects $\left(\dfrac{\Delta q}{\Delta p}\right)$: how demand changes with price
  - **Substitution effect**: change in consumption due to change in relative prices
    * Buy more of the relatively cheaper good, less of the relatively more expensive good
    * Always the same direction, the primary reason for the law of demand (as $p \downarrow, q \uparrow$)
    * Graphically: new bundle of x and y at *new* exchange rate that yields *same* utility as before
      · Shift *new* budget constraint inwards parallel until tangent to original indifference curve
      · Movement from $A \rightarrow B$
  - **Real Income effect**: change in consumption due to change in purchasing power

- ∗ A cheaper good frees up ability to buy more (less) goods overall (and vice versa), despite no change in *nominal* income
  - ∗ Positive for normal goods, negative for inferior goods!
  - ∗ Often smaller than the substitution effect
  - ∗ Larger for goods that are a large portion of budget (e.g. housing, cars, etc)
  - ∗ Graphically: new bundle of x and y at new exchange rate that yields *more* utility than before
    - · Movement from $B \to C$
- – **Total price effect** = substitution effect + real income effect
  - ∗ Graphically: overall movement from $A \to C$
  - ∗ **Law of demand**: $\downarrow p, \uparrow q$

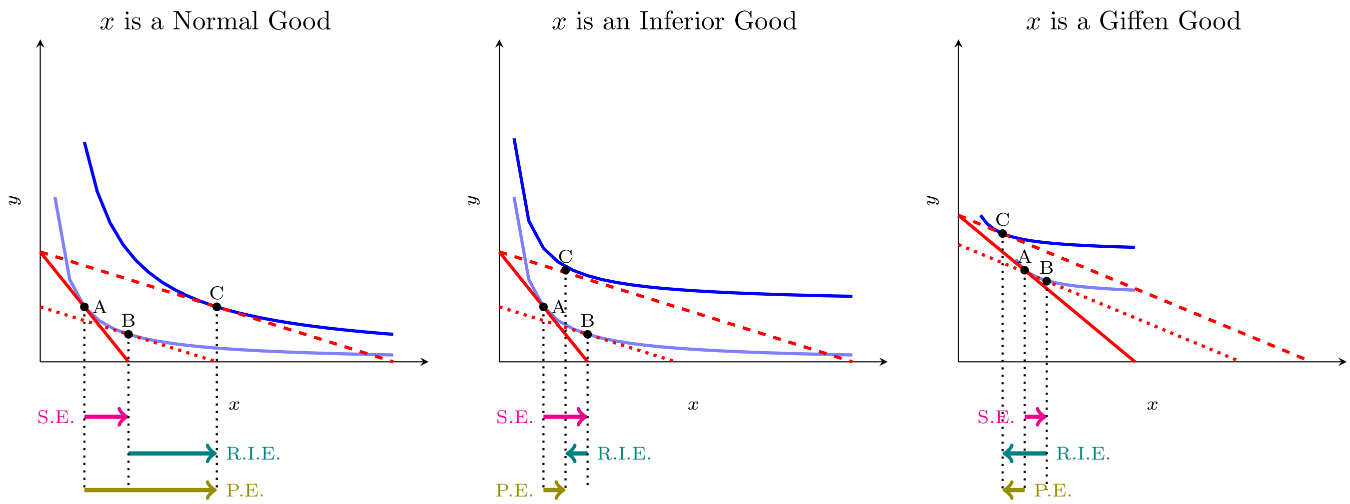


Table 2: Substitution effects ($A \to B$), Real income effects ($B \to C$), and Price effects ($A \to C$) for a decrease in the price of x

- – **Giffen good**: theoretical good that violates law of demand ($\downarrow p, \downarrow q$), requires:
  - ∗ Negative real income effect (an inferior good)
  - ∗ Real income effect > substitution effect (good is a very very large portion of budget)

- Cross-price effects $\left(\frac{\Delta q_x}{\Delta p_y}\right)$: how demand changes with price of *other* goods
  - – **Cross-Price Elasticity of Demand**: how responsive consumption is to changes in price of *another* good

$$\epsilon_{qx,py} = \frac{\%\Delta q_x}{\%\Delta p_y} = \frac{\left(\frac{(qx_2 - q_1)}{qx_1}\right)}{\left(\frac{(py_2 - py_1)}{py_1}\right)}$$

  - ∗ Measures the % change in quantity consumed for a 1% change in price of another good
    - · i.e. "if price of y changes by 1%, quantity of x consumed changes by $\epsilon_{qx,py}$%"
  - ∗ If $\epsilon > 0$: x and y are **substitutes**: $\downarrow p_y, \downarrow q_x; \uparrow p_y, \uparrow q_x$
    - · e.g. Pepsi becoming cheaper reduces demand for Coke (switch to cheaper substitute)
  - ∗ If $\epsilon < 0$: x and y are **complements**: $\downarrow p_y, \uparrow q_x; \uparrow p_y, \downarrow q_x$
    - · e.g. Milk becoming cheaper boosts demand for Cereal (the combination is now cheaper)