# 4.1: Monopoly - Practice Problems (Answers)

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Bob's Bats produces baseball bats, and has the following costs:

$$\begin{aligned} C(q) &= 5q^2 + 720 \\ MC(q) &= 10q \end{aligned}$$

and faces a market demand for bats:

$$q = 120 - 0.4p$$

where quantity is measured in thousands of bats

1. Write Bob's Marginal Revenue function.

**Solution:**

If we can find the inverse demand (of the form $p = a + bx$, we can simply double the slope ($b$) to get marginal revenue. We have the demand function, so solve it for $p$ to get the inverse:

$$\begin{aligned} q &= 120 - 0.4p \\ q + 0.4p &= 120 \\ 0.4p &= 120 - q \\ p &= 300 - 2.5q \end{aligned}$$

This is the inverse demand function, so marginal revenue is

$$MR(q) = 300 - 5q$$

2. Find the profit-maximizing quantity and price.

**Solution:**

First, the quantity, we follow Rule #1 as always: the profit maximizing $q^*$ is where $MR(q) = MC(q)$

$$\begin{aligned} MR(q) &= MC(q) \\ 300 - 5q &= 10q \\ 300 &= 15q \\ 20 &= q^2 \end{aligned}$$

Now that we know the profit-maximizing quantity, we need to find the maximum price consumers are willing to pay for 20 units. Plug this into the inverse demand function:

$$\begin{aligned} p &= 300 - 2.5q \\ p &= 300 - 2.5(20) \\ p &= 300 - 50 \\ p^* &= 250 \end{aligned}$$

3. How much total profit does Bob's Bats earn? Should Bob stay or exit this industry in the long run?

**Solution:**

Total profit again can be found with Rule #2: $\pi = [p - AC(q)]q$

We first need to find the Average Cost function from total cost, by dividing it by $q$:

$$AC(q) = \frac{C(q)}{q} = \frac{5q^2 + 720}{q} = 5q + \frac{720}{q}$$

Now we specifically need to find the average cost at 20 units:

$$
\begin{aligned}
AC(q) &= 5q + \frac{720}{q} \\
AC(20) &= 5(20) + \frac{720}{20} \\
AC(20) &= 100 + 36 \\
AC(20) &= 136
\end{aligned}
$$

Now just plug in the price, average cost, and quantity:

$$
\begin{aligned}
\pi &= [p - AC(q)]q \\
\pi &= [250 - 136]20 \\
\pi &= [114]20 \\
\pi &= 2,280
\end{aligned}
$$

4. At what price would Bob's Bats break even?

**Solution:**

From before, we know that a firm's break even price is at the minimum of its Average Cost curve, where Average Cost is equal to Marginal Cost. First let's find the quantity where that happens:

$$
\begin{aligned}
AC(q) &= MC(q) \\
5q + \frac{720}{q} &= 10q \\
\frac{720}{q} &= 5q \\
720 &= 5q^2 \\
144q^2 & \\
12 &= q
\end{aligned}
$$

This the *quantity* where AC is minimized and equal to MC. We need to find the *price*, so plug this quantity into either AC or MC. MC is easier here:

$$
\begin{aligned}
MC(q) &= 10q \\
MC(12) &= 10(12) \\
MC(12) &= 120
\end{aligned}
$$

The firm breaks even at a price of \$120.

5. How much of Bob's price is markup (over marginal cost)?

**Solution:**

Use the Lerner Index: $L = \frac{p - MC(q)}{p}$. This will tell us what proportion of the price is markup above marginal cost.

First, we do need to find the marginal cost at $q^* = 20$:

$$
\begin{aligned}
MC(q) &= 10q \\
MC(20) &= 10(20) \\
MC(20) &= 200
\end{aligned}
$$

Now plug this and $p^*$ into the Lerner index:

$$
\begin{aligned}
L &= \frac{p - MC(q)}{p} \\
L &= \frac{250 - 200}{250} \\
L &= \frac{50}{250} \\
L &= 0.20
\end{aligned}
$$

The Lerner index says that 20% of the firm's price (\$250) is markup above marginal cost (\$200).

6. Calculate the price elasticity of demand at Bob's profit-maximizing price.

**Solution:**

While you could calculate this manually, it's a lot faster to use the full Lerner Index equation: $L = \frac{p - MC(q)}{p} = -\frac{1}{\epsilon}$. Since we know $L$, we can set it equal to $-\frac{1}{\epsilon}$ and solve for $\epsilon$:

$$\begin{aligned}
L &= -\frac{1}{\epsilon} \\
0.20 &= -\frac{1}{\epsilon} \\
0.20\epsilon &= -1 \\
\epsilon &= -\frac{1}{0.20} \\
\epsilon &= -5
\end{aligned}$$

Demand is elastic. For every 1% the price increases (decreases), consumers will buy 5% less (more).